Sequences and Summation Notation

Infinite Sequence – the sequence has an infinite number of terms.

$a_1, a_2, a_3, a_4, a_5, \dots a_n, \dots$

Finite Sequence – the sequence has a finite number of terms.

$a_1, a_2, a_3, a_4, a_5, \dots a_n$

The first term is denoted a_1, the second term is a_2, etc.

Recursive Formula – when the formula defines the terms using previous term(s).

a_{n-1} is the previous term, a_{n-2} is the term just before the previous term, etc.

Factorial Notation

If n is a positive integer, the notation $n!$ is the product of all integers from n down to 1.

$5! = 5 * 4 * 3 * 2 * 1 = 120$

$8! = 8 * 7 * 6 * 5 * 4 * 3 * 2 * 1 = 40320$

Note: $0! = 1$

To calculate a factorial (without having to type every number in the calculator)

1. Enter the number into the calculator
2. Go to your MATH menu
3. Scroll over to highlight PRB (probability)
4. ! should be one of the choices

Note: Most calculators will not calculate anything bigger than 70!

Summation Notation

$\sum_{i=1}^{n}(a_i) = a_1 + a_2 + a_3 + a_4 + \cdots + a_n$

i is the index of summation, n is the upper limit, 1 is the lower limit

Arithmetic Sequence

A sequence in which each term differs from the preceding term by a constant amount. This constant amount is called the common difference, d.

Examples of Arithmetic Sequences

$142, 146, 150, 154, 158, \dots$	$d = 4$
$-5, -2, 1, 4, 7, \dots$	$d = 3$
$8, 3, -2, -7, -12, \dots$	$d = -5$

Recursive Formula: $a_n = a_{n-1} + d$

General Term of an Arithmetic Sequence: $a_n = a_1 + (n-1)d$

The Sum of the First n Terms of an Arithmetic Sequence: $S_n = \frac{n}{2}(a_1 + a_n)$

Geometric Sequence

A sequence in which each term is obtained by multiplying the preceding term by a constant amount. This constant amount is called the common ratio, r.

Examples of Geometric Sequences

$1, 5, 25, 125, 625, \ldots$	$r = 5$
$4, 8, 16, 32, 64, \ldots$	$r = 2$
$6, -12, 24, -48, 96, \ldots$	$r = -2$
$9, -3, 1, -\frac{1}{3}, \frac{1}{9}, \ldots$	$r = -\frac{1}{3}$

Recursive Formula: $a_n = a_{n-1}r$

General Term of a Geometric Sequence: $a_n = a_1 r^{n-1}$

The Sum of the First n Terms of Geometric Sequence: $S_n = \frac{a_1(1-r^n)}{1-r}$

Geometric Series

A series is an infinite sum. We will use S to designate the series. So we have $S = S_\infty = \sum_{i=1}^{\infty} a_i$.

If $|r| < 1$, r^n will get smaller as n gets bigger. In fact, if n is big enough, r^n is essentially 0. Since we are looking at the case when n is infinity, we can replace r^n with 0. The equation for the sum of a geometric sequence simplifies and we get the equation for a geometric series.

$$S = S_\infty = \frac{a_1(1-0)}{1-r} = \frac{a_1}{1-r}$$

$$S = \frac{a_1}{1-r} \text{ If } |r| < 1$$

Note: If $|r| \geq 1$, then $S = \pm\infty$.